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Modifying Weighted Fuzzy Subsethood-based Rule Models with Fuzzy Quantifiers

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Abstract - **The use of fuzzy quantifiers in linguistic fuzzy models helps to build fuzzy systems that use linguistic terms in a more natural way. Although several fuzzy quantification techniques have been developed, the application of the existing techniques seems very limited. This paper proposes an application of fuzzy quantification to replace crisp weights in subsethood-based fuzzy rule models. In addition to the concern that fuzzy models should have high accuracy rate, attention has also been taken to maintain the simplicity of the generated fuzzy model. The objective is to produce quantifier-based fuzzy models which are not only readable but also practically applicable. The quantifier based fuzzy model is then applied to classification tasks. The classification accuracies of fuzzy models that use crisp weights, continuous quantifiers, multi-valued quantifiers and two-valued quantifiers are compared. Experimental results show that the classification accuracy of the fuzzy model that uses continuous quantifiers is: 1) as good as the classification accuracy of the fuzzy models that use crisp weights, and 2) in most cases, better than fuzzy models that use multi-valued quantifiers or two-valued quantifiers.**

I. INTRODUCTION

Quantification has been regarded as an important topic in fuzzy theory and its applications [1, 3, 4, 5]. The use of fuzzy quantifiers is one way to attach semantic labels to fuzzy sets as with the use of hedges [7]. Fuzzy quantifiers can also be seen as flexible tools for the representation of natural language, making the existing fuzzy models more readable. Although the application of quantifier-based models can be found in the literature [13], such research is relatively limited.

However, several definitions of fuzzy quantifiers have been proposed [1, 3, 4, 5] including so-called absolute quantifiers and relative quantifiers. One of the main issues in the use of fuzzy quantification on fuzzy models is that most of the existing techniques do not deal adequately with practical interests such as *monotonicity*, *antonymy* and *duality* of the quantifiers [1]. This has significantly restricted the take-up of fuzzy quantification techniques.

It is well-known that a particular quantifier that is applicable to a certain model may not be useable in other systems, whereas some other method may be easily adapted to other systems. In particular, crisp weights within the interval of [0,1] may be used to improve the classification accuracy of fuzzy models. Yet it is rather unnatural to modify fuzzy terms with non-fuzzy values. Their use may lead to confusion regarding the semantics of the fuzzy labels and the linguistic interpretation of a given fuzzy system [8]. This paper proposes a method which uses fuzzy quantifiers to modify fuzzy linguistic labels, based on existing fuzzy subsethood-based modelling which attaches crisp weights to each linguistic term in fuzzy rule antecedents [10]. Although there exist different types of quantifier, this paper will mainly refer to the fuzzy relative quantifier Q where $\mu_Q(q) \in [0,1]$ with q defined on real interval [0,1]. In particular, Q possesses the *non-decreasing* behavior: $\forall\ q_1, q_2 \in Q,\ q_1 < q_2 \rightarrow \mu_Q(q_1) \leq \mu_Q(q_2)$. Such a quantifier is based on the quantified statement *"Q Es are As"* where Q is the linguistic quantifier and A and E are fuzzy values defined on $X = \{x_1, x_2, \ldots, x_k\}$. An example of a quantified statement is *"Most students who get a high score are young"*, where *"most"* is the quantifier, *"high"* and *"young"* are the fuzzy values E and A respectively.

The rest of the paper is organised as follows. Section II describes the existing approach of fuzzy linguistic quantifiers and the quantification mechanism. Section III reviews the existing fuzzy modelling techniques which use crisp weights: the Weighted Subsethood-Based Model (WSBA) [10]. Section IV presents the proposed method which applies the reviewed fuzzy quantification technique. Section V presents experimental results. Finally, conclusions and future directions of the research are outlined in Section 6.

II. FUZZY QUANTIFIERS AND QUANTIFICATION

In general, quantifier in logic can be expressed as $Q(x)A(x)$ where $Q(x)$ is a quantifier and $A(x)$ is a predicate for variable x [3]. In classical logic, both the quantifier and the predicate can be represented by crisp sets. In fuzzy logic the quantifier may be apply to crisp or fuzzy sets. A quantifier based on fuzzy sets seems to be more suitable for quantifier based fuzzy models which are described in natural language.

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In general, the membership function $\mu_Q(q)$ of a quantifier Q has no direct meaning. Thus in evaluating a fuzzy quantified proposition, a quantification mechanism is needed to map the membership value $\mu_Q(q)$ such that:

$$F : (\mu_Q(q)) \rightarrow \mathrm{I} \in [0,1] \quad (1)$$

In this paper, the result of evaluating the fuzzy relative quantifier is referred to as the truth-value of the quantifier, and is presented using notation T_Q.

Fuzzy quantification technique can be based on generalization of first order logic quantifiers, where the quantification mechanism involves the definition of the existential quantifier, $\exists$ *(exists at least one)* and of the universal quantifier, $\forall$ *(for all)*. However, the two-valued quantification technique seems too strict as it will return two extreme values thus ignoring the existence of other quantifications that are readily available in fuzzy terms and natural language such as "almost half", "nearly all", "few", "most", etc. Extending this representation language to fuzzy sets, the truth value of the existential relative quantifier and the universal relative quantifier can be defined [1, 3] as:

$$\mathrm{T}_{\exists,A/E} = \mathop{\Delta}_{k=1}^{N}[\mu(e_k)\nabla\mu(a_k)] \quad (2)$$

$$\mathrm{T}_{\forall,A/E} = \mathop{\nabla}_{k=1}^{N}[\mu(e_k) \Rightarrow \mu(a_k)] \quad (3)$$

where $\mu(a_k)$ and $\mu(e_k)$ are the membership functions of fuzzy sets A and E respectively, and $\Rightarrow$ denotes fuzzy implication.

It is obvious that this definition covers as its specific cases classical existential and universal quantifiers. The multi-valued fuzzy quantification can be defined using any available functions such as *non-decreasing*, *non-increasing* or *unimodal* whithin the above definition. Figure 1 shows an example of five different types of fuzzy quantifier that can be defined between the existential quantifier and the universal quantifier. The quantifiers which are labeled Q_1, Q_2, Q_3, Q_4 and Q_5 are interpreted as predefined quantifiers which represent "almost all of them", "almost three-quarter of them", "almost half of them ", "almost a quarter of them" and "a few of them". Several existing methods proposed and discussed in [1, 3] can be used in evaluating the quantified proposition.

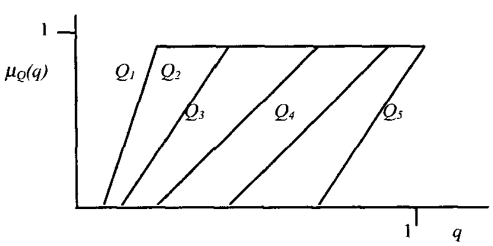

Fig. 1. Example of multi-valued quantifiers

The multi-valued quantifiers can be expanded further because the number of quantifiers that can exist may not be limited to only a few specific ones. However, the problems in expanding this kind of quantifier lie in the need to pre-define each of the quantifiers. Limited pre-defined quantifiers are difficult to be adapted to suit fuzzy models which generate rules based on training data. This is because small changes in the dataset might cause the change of the entire ruleset. Thus, a continuous fuzzy quantification method may be more appropriate.

Vila et al. [12] proposed a continuous quantifier which uses linear interpolation between the two extreme cases of the existential quantifier $\exists$ and the universal quantifier $\forall$. In particular, the quantifier was defined as a linear interpolation:

$$Q(E,A) = (1-\lambda_Q).\mathrm{T}_{\forall,A/E} + \lambda_Q.\mathrm{T}_{\exists,A/E} \quad (4)$$

where Q is the quantifier for fuzzy set A relative to fuzzy set E and λ_Q is the *degree of orness* of the two extreme quantifiers. The truth values of the existential quantifier $\mathrm{T}_{\exists,A/E}$ and the universal quantifier $\mathrm{T}_{\forall,A/E}$ were defined as:

$$\mathrm{T}_{\exists,A/E} = \mathop{\Delta}_{k=1}^{N} \mu(a_k)\nabla\mu(e_k) \quad (5)$$

$$\mathrm{T}_{\forall,A/E} = \mathop{\nabla}_{k=1}^{N} (1-\mu(e_k))\Delta\mu(a_k) \quad (6)$$

where a_k and e_k are the membership functions of fuzzy sets A and E respectively, ∇ represents the *t-norm* and Δ represents the *t-conorm*. This definition will enable the creation of all possible quantifiers that exist between the existential and universal quantifiers.

III. Weighted Subsethood-Based Algorithm (WSBA)

Weighted Subsethood-Based Algorithm (WSBA) [10] is a fuzzy rule induction method based on fuzzy subsethood values. Fuzzy subsethood values represent the degree to which a fuzzy set is a subset of another fuzzy set. For example, for two fuzzy sets A and E, fuzzy subsethood values [2, 14] of fuzzy set A to fuzzy set E, denoted $S(E,A)$ can be defined as follows:

$$S(E,A) = \frac{\sum_{x \in U} \nabla(\mu_E(x), \mu_A(x))}{\sum_{x \in U} \mu_E(x)} \quad (7)$$

where $S(E, A) \in [0,1]$ and ∇ denotes a t-norm operator.

WSBA consists of four main steps:

1. Divide data set into subgroups with respect to the underlying classification outcomes;
2. Calculate fuzzy subsethood values (based on definition (7)) for each linguistic term in each subgroup;
3. Calculate relative weights based on fuzzy subsethood values using the following definition:

$$w(E,A_i)=\frac{S(E,A_i)}{\max_{j=1..l} S(E,A_j)} \quad (8)$$

where $w(E,A_i)$ is the relative weight for linguistic term A_i with regard to classification E; and

4. Create fuzzy general rules based on relative weights.

The WSBA general rules will be in the form of:

Rule 1 **IF** A is ($w(E_1,A_1)A_1$ *OR* ($w(E_1,A_2)A_2$ *OR* ...*OR* $w(E_1,A_i)A_i$) **AND** B is ($w(E_1,B_1)B_1$ *OR* $w(E_1,B_2)B_2$ *OR*... *OR* $w(E_1,B_j)B_j$) **AND** ... **AND** H is ($w(E_1,H_1)H_1$ *OR* $w(E_1,H_2)H_2$ *OR* ... *OR* $w(E_1,H_k)H_k$) **THEN** the class is E_1

Rule 2 **IF** A is ($w(E_2,A_1)A_1$ *OR* ($w(E_2,A_2)A_2$ *OR* ...*OR* $w(E_2,A_i)A_i$) **AND** B is ($w(E_2,B_1)B_1$ *OR* $w(E_2,B_2)B_2$ *OR*... *OR* $w(E_2,B_j)B_j$) **AND** ... **AND** H is ($w(E_2,H_1)H_1$ *OR* $w(E_2,H_2)H_2$ *OR* ... *OR* $w(E_2,H_k)H_k$) **THEN** the class is E_2

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Rule n **IF** A is ($w(E_n,A_1)A_1$ *OR* ($w(E_n,A_2)A_2$ *OR* ...*OR* $w(E_n,A_i)A_i$) **AND** B is ($w(E_n,B_1)B_1$ *OR* $w(E_n,B_2)B_2$ *OR*... *OR* $w(E_n,B_j)B_j$) **AND** ... **AND** H is ($w(E_n,H_1)H_1$ *OR* $w(E_n,H_2)H_2$ *OR* ... *OR* $w(E_n,H_k)H_k$) **THEN** the class is E_n

(9)

In the above definition, 'OR' and 'AND' are fuzzy logical operators and are interpreted by *t-conorm* and *t-norm* respectively. Weights created from fuzzy subsethood values work as multiplication factors for each linguistic term. During the generation, the ruleset is simplified as any linguistic terms that have a weight equal to 0 is automatically removed from the fuzzy rule.

The significant advantage of this method compared to previous subsethood-based methods is the simplicity in generating the fuzzy ruleset. This method does not require any threshold value and generates a fixed number of rules according to the number of classification outcomes.

WSBA has been tested using typical benchmark datasets, including the Iris Plant dataset, and it has the ability to produce high classification accuracy compared to the previous subsethood-based algorithm [10]. However, the multiplication of weight (which is a crisp value) with linguistic term (which is a fuzzy set) may create confusion in the interpretation of such an operation.

IV. Fuzzy Quantifiers for WSBA

The aim of this proposed technique is to replace crisp weights in WSBA by fuzzy quantifiers. In so doing, the quantification method originally proposed by Vila et al. is employed here. Several reasons have been taken into account to support the use of Vila et al.'s approach:

a) The use of *degree of orn*ess enables the implementation of continuous quantifiers. Thus, any possible quantifier can be created in principle.
b) The relative quantifier based method proposed by Villa et al. can be adapted into WSBA easily, thanks to the structure of the WSBA general rule. Thus, the simplicity of WSBA can be preserved.
c) Relative subsethood values can be used as the *degree of* orness *(λ_Q)* of the fuzzy quantifiers. Thus, the two seemingly separate approaches are unified.
d) This approach fulfils the desirable monotonicity and duality properties of quantification [1].

For simplicity, the new algorithm is called as QSBA. The resultant ruleset created by QSBA is as follows:

Rule 1 **IF** A is ($Q(E_1,A_1)A_1$ *OR* ($Q(E_1,A_2)A_2$ *OR* ...*OR* $Q(E_1,A_i)A_i$) **AND** B is ($Q(E_1,B_1)B_1$ *OR* $Q(E_1,B_2)B_2$ *OR*... *OR* $Q(E_1,B_j)B_j$) **AND** ... **AND** H is ($Q(E_1,H_1)H_1$ *OR* $Q(E_1,H_2)H_2$ *OR* ... *OR* $Q(E_1,H_k)H_k$) **THEN** the class is E_1

Rule 2 **IF** A is ($Q(E_2,A_1)A_1$ *OR* ($Q(E_2,A_2)A_2$ *OR* ...*OR* $Q(E_2,A_i)A_i$) **AND** B is ($Q(E_2,B_1)B_1$ *OR* $Q(E_2,B_2)B_2$ *OR*... *OR* $Q(E_2,B_j)B_j$) **AND** ... **AND** H is ($Q(E_2,H_1)H_1$ *OR* $Q(E_2,H_2)H_2$ *OR* ... *OR* $Q(E_2,H_k)H_k$) **THEN** the class is E_2

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Rule n **IF** A is ($Q(E_n,A_1)A_1$ *OR* ($Q(E_n,A_2)A_2$ *OR* ...*OR* $Q(E_n,A_i)A_i$) **AND** B is ($Q(E_n,B_1)B_1$ *OR* $Q(E_n,B_2)B_2$ *OR*... *OR* $Q(E_n,B_j)B_j$) **AND** ... **AND** H is ($Q(E_n,H_1)H_1$ *OR* $Q(E_n,H_2)H_2$ *OR* ... *OR* $Q(E_n,H_k)H_k$) **THEN** the class is E_n

(10)

where $Q(E_n,A_j)$ are fuzzy quantifiers.

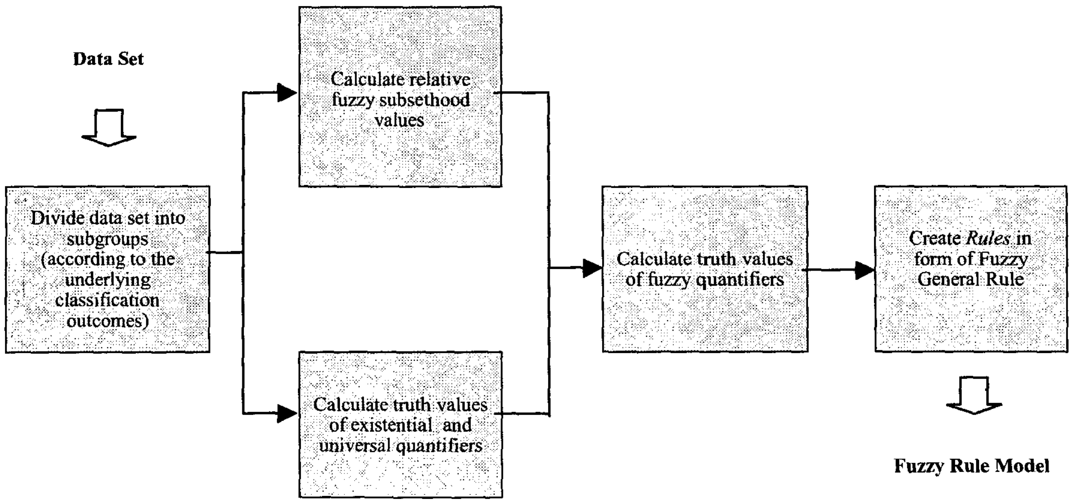

Fig. 2. Framework of QSBA

The crisp weights that were used in WSBA are now replaced by fuzzy quantifiers. Based on the definition of the existential quantifier (5), the universal quantifier (6), and the fuzzy subsethood value (7), it can be proved that if λ_Q is equal to 0 then the truth-value of quantifier Q will also equal 0. Thus any linguistic terms which have the truth-value of the quantifier equal to 0 will be removed automatically from the fuzzy rule antecedents. The final QSBA ruleset will contain the same antecedents as the WSBA ruleset. It is clear that the main structure of WSBA general rules has been preserved. Figure 2 shows the framework of this approach.

The main difference of QSBA compared to WSBA lies in the interpretation of the inference between weights/quantifiers with the linguistic terms. In WSBA the weights for each linguistic term are crisp values and behave as multiplication factor for the linguistic terms. Clearly the use of crisp value will limit the interpretation of $(w_{A_{ij},E_k} \times \mu_{A_{ij}}(x))$. In QSBA both the quantifiers and the linguistic terms are fuzzy sets. This offers flexibility as it enables the use of *t-norm* operators to interpret $\nabla(Q_{A_{ij},E_k}, \mu_{A_{ij}}(x))$ whilst guarantee that the inference results are fuzzy sets.

The use of fuzzy quantifier in QSBA also enables representation of the ruleset in more natural way. This can be shown by the following example:

In WSBA, the ruleset is in the form of "IF SL is (SL1 OR 0.09SL2) AND SW is (SW2 OR 0.2SW3) AND PW is (PL1) AND PW is (PW1) THEN the class is Iris-setosa".

In QSBA, the ruleset will be in the form of "IF SL is ((*almost all*)SL1 OR (*a little*)SL2) AND SW is ((*almost all*)SW2 OR (*almost a quarter of*)SW3) AND PW is ((*almost all*)PL1) AND PW is ((*almost all*)PW1) THEN the class is Iris-setosa.

Clearly, the use of fuzzy quantifiers make the model more readable, although bear in mind that the computation still need to be done using real numbers. This method is different from *Fuzzy Modifier* approach, as the process does not involve re-definition of the original membership function, $\mu_{A_{ij}}(x)$, in order to modify the linguistics labels.

V. Experimental Results

Since the improvement in model interpretability is obvious, the objective of the experiments is to compare the classification accuracy obtained by the proposed method (QSBA) with the classification accuracy obtained by the method that used crisp weights (WSBA). The datasets involve in this experiments were obtained from UCI machine learning repository [11], which have been widely used as benchmarks in classification tasks. The datasets are Iris-Plant, Wine Recognition, Glass Identification and Wisconsin Breast Cancer datasets as summarised in Table I.

TABLE I
THE CLASSIFICATION PROBLEMS DATASETS

Name	Number of Instances	Number of Inputs	Number of main classes
Iris-Plant	150	4	3
Wine Recognition	178	13	3
Glass Identification	214	9	2
Wisconsin Breast Cancer	699	9	2

Ten-fold cross-validation was used to evaluate the classification accuracy: Each dataset was divided randomly into ten subsets, with nine subsets used for training and the remaining one for testing. Thus, ten sub-experiments have been carried out for each dataset.

The fuzzy models involved in these experiments are induced using four different methods:

a) WSBA: fuzzy subsethood-based model with crisp weight,
b) QSBA(1): fuzzy subsethood-based model with continuous fuzzy quantifier,
c) QSBA(2): fuzzy subsethood-based model with multi-valued quantifier, and
d) QSBA(3): fuzzy subsethood-based model with two-valued quantifier.

In all of the experiments, a simple fuzzification method based on triangular and trapezoidal membership functions was used to transform the crisp values into fuzzy values. The *Min-Max* operator was used as the fuzzy inference mechanism for all induced rulesets. Note that the membership values of the datasets were not in any way optimised. It is therefore quite possible that rulesets with even better classification accuracies may be induced by these methods if an optimised fuzzification scheme is available.

For experiments using QSBA(2), five pre-defined quantifiers were created to represent the quantifiers "almost all", "almost three-quarters", "almost half", "almost a quarter" and "a little". For experiments using QSBA(3), the quantifiers were created based on a two-valued quantifier "at least *m*" where *m* can be replaced by a term such as a half, a quarter, ten percent, etc. Results of these experiments are shown in Tables II, III, IV and V, giving the best, worst and average classification accuracy over the training and testing dataset.

TABLE II
CLASSIFICATION ACCURACY BASED ON 10-FOLD CROSS-VALIDATION FOR IRIS PLANT DATASET

Fuzzy Models	Dataset	Classification Accuracy (%)		
		Best	Worst	Average
WSBA	Training	97.04	95.56	96.60
	Testing	100	86.67	96.00
QSBA(1)	Training	97.04	96.3	96.67
	Testing	100	86.67	96.00
QSBA(2)	Training	97.78	94.07	96.60
	Testing	100	80.00	94.00
QSBA(3)	Training	95.56	90.37	93.55
	Testing	100	80	91.35

TABLE III
CLASSIFICATION ACCURACY BASED ON 10 FOLD CROSS-VALIDATION FOR WINE RECOGNITION DATASET

Fuzzy Models	Dataset	Classification Accuracy (%)		
		Best	Worst	Average
WSBA	Training	98.14	96.88	97.70
	Testing	100	88.89	96.67
QSBA(1)	Training	98.14	96.88	97.57
	Testing	100	88.89	96.11
QSBA(2)	Training	83.75	76.88	80.53
	Testing	88.24	61.11	74.74
QSBA(3)	Training	85.09	80.63	82.46
	Testing	88.89	70.59	79.15

TABLE IV
CLASSIFICATION ACCURACY BASED ON 10-FOLD CROSS-VALIDATION FOR GLASS* IDENTIFICATION DATASET

Fuzzy Models	Datasets	Classification Accuracy (%)		
		Best	Worst	Average
WSBA	Training	95.85	93.23	94.70
	Testing	100	86.36	93.96
QSBA(1)	Training	95.85	94.27	95.02
	Testing	100	86.36	93.98
QSBA(2)	Training	95.34	89.58	92.83
	Testing	100	80.00	87.33
QSBA(3)	Training	76.56	73.06	74.77
	Testing	90.48	59.09	74.85

* *(classified into window glass and non-window glass)*

TABLE V
CLASSIFICATION ACCURACY BASED ON 10 FOLD CROSS-VALIDATION FOR BREAST CANCER DATASET

Fuzzy Models	Dataset	Classification Accuracy (%)		
		Best	Worst	Average
WSBA	Training	93.64	92.53	93.02
	Testing	98.57	90	93.85
QSBA(1)	Training	94.12	93	93.50
	Testing	97.14	89.86	93.27
QSBA(2)	Training	85.21	83.94	84.69
	Testing	91.43	80	84.73
QSBA(3)	Training	74.88	64.07	70.42
	Testing	78.57	55.71	68.11

The experimental results show that the classification accuracy of the ruleset induced via the use of continuous fuzzy quantifiers, QSBA(1), is as good as the classification accuracy of the model learned by WSBA with crisp weights. The results are indeed consistent in all four datasets in this regard.

The results also show that in most cases, the QSBA(1)model that used continuous fuzzy quantifiers produced better classification compared to the ruleset induced by the use of multi-valued fuzzy quantifiers, QSBA(2), or two-valued quantifiers, QSBA(3).

Comparing QSBA(2) with WSBA, the experiment using the Iris Plant dataset shows that the classification accuracy of the model learned with multi-valued fuzzy quantifiers is as good as the classification accuracy of the model that used crisp weights. However, in the other three experiments, the model that used multi-valued quantifiers did not perform so well. Comparing QSBA(3) with WSBA, the results show that WSBA performs better than QSBA(3), which seem to reflect the drawback of the two-valued quantification approach.

To be objective, it is finally also worth mentioning that in one case in the experiments involving Irish Plant dataset, the model that used multi-valued fuzzy quantifiers outperforms the model that used crisp weights and the model that used continuous fuzzy quantifiers.

VI. CONCLUSION

This paper has presented an application of fuzzy quantifiers in fuzzy subsethood-based models for classification tasks. It has been shown in this study that the adaptation of fuzzy quantification can be done in such manner that the structure of the existing fuzzy model is preserved to maintain the simplicity of the current method. Thus, applying fuzzy quantifiers to a fuzzy model does not necessarily make the existing model more complicated. It has also been shown that fuzzy quantifiers can be used to replace crisp weights used in Weighted Fuzzy Subsethood-based model (WSBA). The experimental results show that the fuzzy model that uses continuous fuzzy quantifier performs as well as the fuzzy model that uses crisp weights, whilst improving the comprehensibility of the model.

It has been shown that the uses of continuous fuzzy quantifiers often produce better results when compared to the use of multi-valued quantifiers or two-valued quantifiers. However, further research should be done to investigate this result as in some cases the model that used multi-valued quantifiers may lead to better classification accuracy. It may also be worth investigating the differences between the generation of fuzzy rules involving expert opinions, where a model that uses pre-defined multi-valued quantifiers may be very useful, and the generation of fuzzy rules involving information from data, where continuous quantifiers may be more appropriate.

Finally, note that careful attention should be taken when adapting existing fuzzy quantification methods into fuzzy models because of the availability of different definitions of the existing fuzzy quantifiers. A generic definition of fuzzy quantifiers to be used as a general quantification mechanism for an arbitrary fuzzy model is not yet available and may be impossible to create. Thus, the use of specific quantification methods for a particular fuzzy models seems unavoidable.

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